Exam Symmetry in Physics

Date	June 30, 2017
Time	14:00 - 17:00
Lecturer	A. Borschevsky

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the lecture notes, nor other notes or books
- The weights of the exercises are given in the table below
- Illegible handwriting will be graded as incorrect
- Good luck!

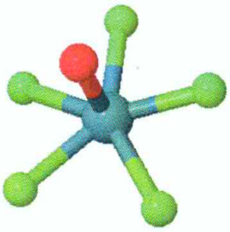
Weighting

1a)	7	2a)	6	3a)	6
1b)	5	2b)	11	3b)	4
1c)	3	2c)	4	3c)	4
1d)	4	2d)	9	3d)	3
1e)	10			3e)	5
1f)	6			3f)	3

$$\text{Result} = \frac{\sum \text{points}}{10} + 1$$

Exercise 1

Consider the $XeOF_5^-$ molecular ion (see figure). Its symmetry group is C_{5v}.



(a) Identify all the symmetry transformations that leave this system invariant and divide them into conjugacy classes.

(b) Show that this symmetry group is isomorphic to D_5.

(c) Determine the number and the dimensions of all the inequivalent irreps of C_{5v}.

(d) Construct explicitly the three-dimensional vector representation D^V for each class of transformations and extract a 2 dimensional irrep from it.

(e) Construct the character table of this group.

(f) Determine the characters of the direct product representation $D^V \otimes D^V$ of this group and use them to determine the number of possible independent invariant tensors (no need to construct them explicitly).

Exercise 2

Consider the point group D_6 (see character table below).

D_6	e	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$
A_1	1	1	1	1	1	1
A_2	1	1	1	1	-1	-1
B_1	1	-1	1	-1	1	-1
B_2	1	-1	1	-1	-1	1
E_1	2	1	-1	-2	0	0
E_2	2	-1	-1	2	0	0

(a) Find 6 (of the total of 15) subgroups of this group, with at least one of them of order 6.

An atom is placed in a crystal of point group D_6.

(b) Check how the $l = 2$ eigenstates of the central potential are split by the crystal potential, giving the new degeneracies. The character of $SO(3)$ for orbital angular momentum l is given by: $\chi^{(l)}(\theta) = 1 + 2(\cos\theta + \cos 2\theta + ... + \cos l\theta)$.

(c) Introduction of a magnetic field along the original 6-fold axis reduces the symmetry further, to C_6. How are the degeneracies affected?

(d) Construct the vector representation D^V of D_6. Decompose it into irreps and use this to conclude whether this group in principle allows for an electric dipole moment.

Exercise 3

(a) Consider the following sets of elements and composition laws. Determine for each of the sets whether it is a group. Explain your answer.

i. Integers, multiplication.
ii. Even integers, addition.
iii. The n-th roots of unity ($e^{2\pi ik/n}$ for $k = 0, 1, .., n-1$), multiplication.

Consider the group of rotations and reflections in 3 dimensions, $O(3)$.

(b) Is this group Abelian? Explain your answer.

(c) Explain what an axial vector is and give an example of one. How does an axial vector transform under $O(3)$?

Now consider the group of rotations in two dimensions, $SO(2)$.

(d) Write down the defining representation of $SO(2)$.

(e) Write down the two-dimensional representation of $SO(2)$ obtained by its action on the vector

$$\begin{pmatrix} x + iy \\ x - iy \end{pmatrix}$$

(f) Are the two above representations equivalent? Explain your answer.